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Radiation Resistance, Feed Point Impedance and Mythology

An understanding of these topics is vital to antenna experimenters, yet many continue to misunderstand the definitions.

It is not surprising that there remains a great deal of confusion about antenna radiation resistance generally, and its relationship to feed point impedance in particular. Many authors state special cases as being general rules. To top it off, some actually state that there are multiple definitions of radiation resistance. We are all free to invent whatever definitions we wish about anything, but such made-up definitions do not relate to the larger body of knowledge and only serve to further confuse an already confusing issue. Many technical publications, including Amateur Radio publications, state false definitions of radiation resistance and erroneous values for a variety of examples.

Of all the world-class authors on the subject, John Kraus provides the most comprehensive discussions in a single text that I have found.[1] He defines radiation resistance from several different approaches and applications. The equations he published for the definition take several different forms in that they use differing variables. Some readers have mistakenly interpreted these multiple definitions as "different" definitions. In this paper I will show that the special case definition for radiation resistance in vertical antennas (of special interest to Amateur Radio operators) can be derived from the general case definition. Kraus does not provide these derivations, but the book is written as a teaching text. As such, one can imagine such derivations were assigned as graduate-level homework assignments, and perhaps even Masters' thesis topics.

[1]Notes appear on page 35.

By working through this derivation we can develop a deeper understanding of this very difficult yet critically important antenna parameter. Further, we can converge on an unambiguous definition of radiation resistance, using both a general equation and a verbal description.

For purposes of simplicity this paper will focus upon linear in-line antennas, i.e. single elements using a straight conductor that has a diameter very small compared to the wavelength. Multi-element arrays, "bent" arrays, planar structures and 3-d antennas can be very complex. NEC-based modeling tools can approximate multi-element antenna radiation resistance if used carefully. E&M modeling tools become indispensable for 2 and 3D structures. The methods and definitions in this paper, however, can for a basic understanding, necessary for calculation of Rr in more complicated antenna arrays.

Basic Concepts

"Radiation resistance," R_r, is the result of the antenna coupling (losing power by radiating) RF power into a medium, usually "free space." Whenever power is dissipated or "lost" a resistance is involved. In any real antenna there are also resistive losses that are not part of the radiation resistance. The power lost to ohmic resistance is dissipated as heat, not as "radiation" and is usually designated as R_L, or "loss resistance." It is relatively easy to visualize loss resistance in antenna elements (wires or tubing) or the ground. More difficult to conceptualize is "losing" power to space.

If "space" "accepts" RF energy by providing a medium for that power, it must have some type of impedance. An analogy is the transmission line. When the line is matched (SWR = 1:1) then the voltage and current are in phase and their ratio is equal to the characteristic impedance of the line. This is just Ohm's Law. If we have an infinite transmission line, it will accept RF power and appear as a pure resistance, yet there is no resistance (in a perfect line). We can think ofZ_0 (the characteristic impedance of free space) comparable to Z_0 (the characteristic impedance of a transmission line).

In any calculation involving impedance, Ohm's Law and the power law can be applied. $P = IE$, and/or $Z = E / I$. When electromagnetic energy is radiated into space there are magnetic and electric field components of the wave. These fields are measured in volts/meter and amperes/meter. So, the characteristic impedance of free space is: $Z_0 = V_m / I_m$. The "meter" terms cancel so we are left with a simple Ohm's Law calculation. This ratio of the electric and magnetic field values is a result of the permittivity,ε_0 and permeability, μ_0 of free space, thus Z_0 is also =

$$\sqrt{\frac{\mu_O}{\varepsilon_O}}$$

As an aside, the speed of light through any medium is:

$$c = \sqrt{\frac{1}{\mu\varepsilon}}\,.$$

Obviously the speed of light is intrinsically related to R_r. Furthermore, the speed of light and the impedance of *any* medium are also based on these two related equations. In free space and the far field of the antenna, the ratio of values of the electric and magnetic field are *always* constant. Again, the ratio is defined by Z_0 = 377 Ω, often abbreviated as 120 π Ω.

The calculation of Z_0 of a transmission line using air (free space) as its dielectric is given by Equation 1.

$$Z_0\left(\text{coax}\right) = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, ln\frac{D}{d} \qquad \text{[Eq 1]}$$

where D is the inside diameter of the coaxial shield and d is the outside diameter of the internal conductor. Thus, the characteristic impedance of an air core transmission line can be calculated the same way as the characteristic impedance of free space, with the difference being that the electric and magnetic fields are "trapped" between the two conductors. Inside a transmission line the waves are "guided," in free space they are "radiated."

Radiation Resistance Defined

We can approach deriving a general equation for R_r by using several different methods. In this paper I will attempt to show that the value of R_r given by different equations by Kraus, in effect, reflect the same definition.

Let's begin with a simple method to form an intuitive understanding; the power law. In principle, we can find any value of R by Ohm's Law as above or by the power law: $R_r = P / I^2$.

Imagine an isotropic transmitting antenna in free space. Surrounding the antenna of interest is a large imaginary sphere. The radius of the sphere is large compared to the wavelength of operation (in the far field of the antenna). The sum of all *power* propagating through this sphere is the same as the total power radiated by the antenna. The RF *current* at a *maximum* point along the antenna length represents the current value we use in the power law equation. Thus if we know the total power radiating away from the antenna in free space and the current at a maximum point along the antenna, we can directly calculate the radiation resistance using Equation 2.

$$R_r = P / I^2 \qquad \text{[Eq 2]}$$

This is the very simple and intuitive general form equation for radiation resistance for an isotropic antenna.

If the antenna is lossless, then these two terms will exactly define R_r. Since losses (usually series losses) exist, however, the current will be a bit higher for a given radiated power than R_r. The radiation resistance can never be equal to the feed point impedance because of losses (unless you use a super-conducting antenna). The same thing can be said about a pure reactance, however. That should not imply that we despair and forget about a proper definition for X_c, X_l, or R_r!

For non-isotropic antennas, we need a bit of refinement. As a general definition, Kraus defines R_r as given by Equation 3.

$$R_r = \frac{S(\theta,\phi)_{max}\, r^2 \Omega_A}{I^2} \qquad \text{[Eq 3]}$$

This is the general-form equation for radiation resistance. Two of the three key terms we have already mentioned: power and current, so this is really just a special expression of the power law ($R = P / I^2$, but with some important subtleties. As in the simple case given earlier, the power term is in the numerator. For non-isotropic cases, however, the term $S(\theta,\phi)_{max}$ is a necessary refinement of the simple isotropic solid angle of 4π, a field power density (called the Poynting vector) over the spherical coordinates (like longitude and latitude). In other words, the power term is now the sum of all the power propagating through *a portion* of the imaginary sphere instead of the entire sphere we discussed earlier. In this case, Kraus is using the point that is the maximum power point on the sphere's surface — more on this later. In the isotropic case, as above, all points on the sphere have equal power density, so Equation 3 simplifies to the simpler power equation.

Moving through the numerator, the sphere has a radius of r, and then there's a possibly confusing term, **Ω**. Usually **Ω** designates "Ohms," but not in this case. Here, **Ω** designates a solid angle, which in turn defines a portion of the sphere's surface (like the Pacific Ocean defines a portion of the surface of the spherical earth). Both ohms and solid angles are needed in this paper, so I'll follow a convention that uses bold font for the solid angle term. With solid angles, 4π defines the entire sphere, 2π a hemisphere, and so on. So, an isotropic antenna will radiate with a pattern of 4π, equal power in all directions. Then Ω defines the portion of the sphere that is the 3 dB beamwidth area surrounding the point of maximum power propagating through the sphere, or $S(\theta,\phi)_{max}$. The RF current squared term, I^2, appears on the antenna element at a *current maximum*.

So we see in this general equation that R_r is a function of three fundamental terms — total power radiated, antenna current, and the beamwidth of the antenna pattern. Most readers will recognize that beamwidth is also a function of antenna gain, where an isotropic antenna has an aperture (gathering area) of $\lambda^2 / 4\pi$. Gain is proportional to aperture, so an antenna with 3 dBi gain will have an aperture of $2\lambda^2 / 4\pi$, since 3 dB is a power difference of 2. An isotropic antenna has a solid angle, **Ω**, of 4π, therefore, 4π **Ω** = 4π / G, where G is the power gain, in this case 1, assuming no loss. Thus, the higher the gain of an antenna, the greater the antenna's aperature and the smaller the antenna's solid angle defining the 3 dB beamwidth.

From an intuitive view, imagine that it is "more difficult" to radiate into a smaller portion of free space than the full sphere of free space, so the smaller the solid angle (**Ω**) the higher the gain, and the lower the R_r.

An example to illustrate this effect is to compare a "perfect" ground mounted ¼ λvertical antenna over a perfect conductive ground (R_r = 36 Ω) and a ½ λ dipole in free space (R_r = 73 Ω). The maximum broadside gain of the vertical is 5.14 dBi and the gain of the dipole is about 2.14 dBi, about 3 dB difference, or a power gain difference of exactly 2. The R_r difference is also 2, indicating a linear relationship between antenna power gain, antenna aperature, **Ω** and R_r. Additionally, it is easy to imagine that the vertical antenna is only radiating into one hemisphere here, defined by the hemisphere above the ground plane, and the dipole is radiating into both hemispheres (no ground to divide free space), Thus the solid angle, **Ω**, is also half for the vertical compared to the dipole.

Equating Kraus' Radiation Resistance Equations

Of special importance for radio amateurs is the value of R_r for vertical antennas. In a previous paper, I provided a detailed explanation of R_r for vertical antennas.[3] For vertical antennas Kraus gives the following equations.

$$A_e = \frac{h_e^2 Z_o}{4R_r} \qquad \text{[Eq 4]}$$

Rearranging terms we get the radiation resistance for a vertical antenna:

$$R_r = \frac{h_e^2 Z_0}{4A_e} \qquad \text{[Eq 5]}$$

Where A_e is the antenna aperture, measured in m^2 (directly proportional to gain)h_e is the antenna height measured in meters, where:

$$h_e = \frac{I_{ave}}{I_0} h_p \qquad \text{[Eq 6]}$$

where I_{av} is the *average* current along the vertical antenna element, I_0 is the *maximum* current along the antenna, and h_p is the actual physical length of the vertical. The impedance of free space, Z_0, is measured in ohms, and R_r is the radiation resistance, also measured in ohms. Here, the terms containing linear dimensions cancel and we are again left with ohms.

Any definition of radiation resistance (defined by ohms) *must* yield only ohms, unless you want to redefine other terms as well to make your equation work! Valid equations defining radiation resistance simply substitute other terms that, in turn, must be valid. Let's see if the Kraus equation for the general case (Equation 3) can be shown to be the same as the particular case for a vertical (Equation 5).

We can answer this question by substituting terms for their equivalents. We will assume an isotropic case for both, thus Equation 3 becomes Equation 7.

$$R_r = \frac{S(\theta,\phi)_{max} 4\pi}{I^2} \quad \text{[Eq 7]}$$

where $4\pi = \mathbf{\Omega}$ for an isotropic antenna and Equation 5 becomes Equation 8.

$$R_r = \frac{4\pi h_e^2 Z_0}{4\lambda^2} \quad \text{[Eq 8]}$$

or

$$R_r = \frac{\pi h_e^2 Z_0}{\lambda^2} \quad \text{\{Eq 8A]}$$

where $\lambda^2 / 4\pi$ is the aperture of an isotropic antenna, or A_e. Thus we have assumed the isotropic case for both equations.

If Equations 7 and 8 can be shown as equal, then R_r has the same definition for the general case and the special case for the vertical antenna. Again, for the isotropic case, $S(\theta,\phi)_{max}$ represents the total power radiated by the antenna. Kraus also shows that the total radiated power can be defined as the value of the square of the radiated magnetic field: $H^2 = S(\theta,\phi)_{max}$, where

$$H = \frac{\sqrt{Z_0}\, 2\pi f I h_p}{4\pi c}$$

and where c is the speed of light. This is simply another form of the power law, where power is a function of the current squared, and a magnetic field strength is directly proportional to the current creating the field.

Therefore, radiated power is

$$H^2 = Z_0 \frac{4\pi^2 I^2 h_p^2}{16\pi^2 \lambda^2},$$

or

$$H^2 = \frac{Z_0 I^2 h_p^2}{4\lambda^2}$$

Now we can multiply by 4π ($\mathbf{\Omega}$) (for the isotropic case) and divide by I_0^2 to derive the following equations.

$$R_{r=} \frac{Z_0 I^2 h_p^2}{4\lambda^2 I_0^2} 4\pi \quad \text{[Eq 9]}$$

or

$$R_{r=} \frac{Z_0 \pi I^2 h_p^2}{\lambda^2 I_0^2} \quad \text{[Eq 10]}$$

or

$$R_{r=} \frac{Z_0 \pi h_e^2}{\lambda^2} \quad \text{[Eq 11]}$$

Equation 11 is R_r derived from the general equation for an isotropic antenna. Now let's derive R_r from Kraus's special equation for vertical antennas (Equation 5). Again for the isotropic case, substituting $\lambda^2 / 4\pi$ for A_e, we have Equation 12.

$$R_r = \frac{4\pi h_e^2 Z_0}{4\lambda^2} \quad \text{[Eq 12]}$$

or

$$R_r = \frac{Z_0 \pi h_e^2}{\lambda^2} \quad \text{[Eq 13}$$

Equation 13 is identical to Equation 11 as long as the $\mathbf{\Omega}$ coincides with the value for A_e as explained above. If we change the gain from isotropic, both equations simply change value by identical coefficients as described earlier. (The isotropic case simplifies the derivation considerably, however).

Thus Kraus presents only one definition of R_r. Furthermore, we can derive R_r by working from an integration of the radiated output power together with *maximum* antenna current and the gain of the antenna (Equation 3), or we can derive R_r from the *distribution* of current on the antenna, the *maximum* current on the antenna, and the gain of the antenna (special case for the vertical antenna, Equation 5). Thus we can see that R_r is a function of all these terms. It all depends on using the proper terms to set up more convenient equations for specific applications. Like other terms used in Physics, R_r is a well-defined term that can be derived using standard equations, including the most fundamental equations of electromagnetic science: Maxwell's Equations.

Radiation Resistance and Feed Point Impedance

The impedance at an antenna's feed point depends upon the frequency of operation, physical characteristics of the antenna, the current distribution, its relationships to objects, the impedance of free space, and the *point* on the antenna where the power is applied. All these conditions result in a ratio of voltage and current (the real part of the feed point impedance) and the phase relationship between voltage and current (the reactive part of the feed point impedance).

Therefore, the same dependencies that determine the feed point impedance also affect the radiation resistance but the calculations to derive the two terms use different equations because they are not identical. I will attempt to offer a non-mathematical description of the necessary conditions for feed point impedance to equal radiation resistance (assuming no loss).

Thus far I have hinted at a basic relationship between radiation resistance and feed point impedance — at a current maximum along an antenna. This is an important first step, but we need some refinement.

If we measure the feed point impedance (resistance and reactance) as purely reactive (no real part of the impedance) then there is no power loss and thus no radiation. Of course there is *always* some loss in real antennas or circuits. If there is a resistive portion of the feed point impedance then power is being lost as either heat (conductor loss) and/or radiation. The feed point impedance and radiation resistance are *never* equal because there is *always* resistive loss (with the unlikely exception of using a superconductor antenna). In the case of an antenna, where the usual desired effect is to minimize loss and maximize power transfer to (or extraction from) free space we can express this often published relationship as antenna *efficiency*.

$$Eff = \frac{R_r}{R_r + R_l} \quad \text{[Eq 14]}$$

Eff is the antenna efficiency, R_r is the radiation resistance and R_l is the ohmic resistance resulting in power dissipated by heat.

Mythology

Myth #1: Radiation resistance is a "part" of the feed point impedance.

This is true only in specific cases and is a major source of confusion as a general definition. For example, if we center feed a ½ λ resonant dipole in free space, the feed point impedance is about 73 Ω of pure resistance. The radiation resistance is also about 73 Ω. The feed point impedance will probably be measured at a bit higher value than 73 Ω because of ohmic losses (heat) in the antenna. If the ohmic losses are 1 Ω, then the feed point impedance would be 74 Ω, and by Equation 14, the antenna efficiency would be about 98.6 %. In this special case, Myth #1 is true. In this special case, the transformation of source impedance (feed point) to load impedance (radiation resistance) is 1:1.

If we feed the dipole off center, however, let's say at ¼ of the distance from one end instead of half way, the feed point impedance is 138 Ω real but the radiation resistance is still 73 Ω and the ohmic resistance remains very small. If Myth #1 were true, then we have 65 Ω of unaccounted resistance. The ohmic losses are still only due to about 1 Ω, so subtracting 73 from 138 is meaningless. The feed point impedance has simply been "transformed" by moving it off the point of current maximum along the dipole. Since the values of current and voltage change along the length of an antenna, antenna elements can become impedance transformers for feed points. In the case of a ½ λ dipole, however,

R_r remains constant no matter where the feed point is placed along the antenna, but the feed point impedance changes dramatically with changing the feed point location.

Myth #2: Radiation resistance is equal to feed point impedance plus losses in a center fed antenna.

Again, as above, this is true only in a special case. Consider a center-fed folded ½ λ dipole. The feed point impedance is about 300 Ω, but the radiation resistance remains the same as a single-wire dipole, about 73 Ω. Assuming that a folded antenna has 4× the radiation resistance of a single conductor antenna is a common error. Antenna elements (as well as transmission lines) can also behave as transformers as in the cases of folded antennas, while terms defining radiation resistance remain constant. This is also another obvious case where Myth#1 is false. (As an aside, folded dipoles exhibit lower *Q* than a single-wire counterpart, making them more broad-banded).

The separation of the two conductors in a folded dipole is assumed to be a very small fraction of a wavelength. The currents flowing on adjacent points of the two conductors simply add when forming the radiation wave. If the currents are in phase and equal (another assumption of the folded dipole), the effective current is doubled (as far as radiation is concerned), but the feed point is connected to only one conductor. This result is the feed point current is ½ the total effective current at that point, resulting in a 4× increase in feed point impedance, but, again, R_r remains constant at 73 Ω.

Another example is a two-element collinear antenna, which is actually a full-wavelength dipole fed in the center. The radiation resistance increases to near 100 Ω, but the feed point impedance is over 1000 Ω. Again, there is no direct relationship between feed point impedance and radiation resistance.

Myth #3: The feed point impedance of a base-fed vertical is radiation resistance plus the antenna losses.

This is only true for single conductor verticals that are electrically ¼ λ or shorter. A perfect ¼ λ vertical over a perfect ground will have a radiation resistance of about 36 Ω, the same as the feed point impedance.

Now let's place a capacitance hat on the ¼ λ vertical, which also has an equivalent electrical length of ½ λ. Instead of the current maximum appearing at the base, the current maximum is now at the top of the vertical. The radiation resistance remains the same at 36 Ω, but the feed point impedance is over 1000 Ω. So much for using the base fed vertical myth.

As the vertical is made longer than ¼ λ, the feed point impedance is no longer the same as the radiation resistance. This is most dramatically shown for a ½ λ (actual height) vertical whose feed point impedance is over 1000 Ω of real impedance value, yet the radiation resistance is about 100 Ω.

Let's look at another example: a folded ¼ λ vertical. In this case we have a two ¼ λ wires closely spaced and shorted at the top. One wire is fed against the ground, the other is connected to ground, thus appearing to be a folded dipole with the ground acting as counterpoise. The radiation resistance again remains the same 36 Ω, but the feed point impedance is now 144 Ω. Again we see an impedance transformation, but no effect on radiation resistance.

Myth #4: The feed point impedance is equal to the radiation resistance plus losses only at a current maximum on the antenna.

This is getting closer to a correct correlation, but the examples of both the horizontal and vertical folded antennas prove this general statement to be untrue. We can now define a set of practical conditions (especially for most amateur work), however, where the feed point impedance actually equals the radiation resistance.

Relationship Between Radiation Resistance and Feed Point Impedance: The real portion of the feed point impedance equals the radiation resistance plus losses of the antenna only for single-conductor antennas fed at a current maximum along the antenna.

The feed point will coincide with a current maximum at the center of a balanced antenna that is less than or equal to an electrical ½ λ long. It will also coincide with the center of a horizontal antenna that is an odd number times ½ λ.

For base-fed vertical antennas, the feed point will be at a current maximum when the electrical length of the vertical is less than or equal to ¼ λ or odd multiples of an electrical ¼ λ.

For other situations, intuition easily breaks down and an analytical tool becomes invaluable. Current maximums are conveniently illustrated in many antenna simulation software tools. For example, *EZNEC* shows current values along conductors in all of its simulations. Therefore, if your feed point is located at a current maximum, the real portion of the feed point impedance will be the simulated radiation resistance plus losses. Any statement equating radiation resistance plus losses and feed point impedance should not appear as "general rule" statements but rather include a brief description of "why" for some set of special cases.

Another complexity: In antennas longer than ½ λ the current distribution (and thus the radiation resistance) can be changed by changing the feed point location. So, when calculating and/or measuring the location(s) of current maximum(s) along an antenna element, be careful that key terms that define R_r are often changed by changing the feed point position.

In amateur applications, radiation resistance is most often important in vertical antenna installations, especially when the vertical is shorter than ¼ λ, and especially critical in HF mobile installations. In these cases the above definition does indeed apply. The common mistake, however, is to apply the definition to a more general case, which usually leads to mistakes. In almost every amateur vertical antenna installation, loses will be series ground losses. In mobile low-band antennas, however, the conductor losses of the antenna proper may also play a part as the radiation resistance may be *milliohms*.

For a much deeper understanding of the terms used and derivations presented in this paper the reader is invited to read the three references given. The Kraus text develops the terms and formal proofs using advanced mathematics, especially integral and vector calculus. In two earlier *QEX* articles, I attempted to simplify the complexity needed to quantify antenna theory, in this case the Kraus text.[2,3] This paper, in turn, focuses specifically on a deeper treatment of radiation resistance and the often-confused relationship between radiation resistance and feed point impedance deriving fundamental theory and derivations from the three references.

Bob was first licensed in 1966 at age 15. He is an ARRL Life Member and Technical Advisor. He is currently a senior RF engineer for Trimble Navigation, working in Research and Development for advanced GPS and RFID systems, including antenna design. He has a BS in Physics from the University of Oregon and has published more than 60 papers and articles in professional and amateur publications on RF engineering topics and has been awarded 6 patents.

Bob's current Amateur Radio interests include low-band and 6 meter DXing, with 9BDXCC (160-10 meters), DXCC Honor Roll, accomplished using only tree-supported wire antennas, CWDXCC Honor Roll, 5BWAZ. Beside an Elecraft K3 transceiver and a few accessories Bob's entire station is homebrew, including the first tower he ever owned. He published the first conceptual diagram of an SDR in 1988, published the first paper describing the use of DDS in FM broadcast exciters, dramatically improving the linearity of analog FM stereo and now the industry standard, designed the first MOSFET-ring RF mixer (Si-8901), as well as numerous other contributions.

Notes

[1]John D. Kraus, W8JK, *Antennas*, 1988, McGraw-Hill.

[2]Robert J. Zavrel Jr, W7SX, "How Antenna Aperture Relates to Gain and Directivity," *QEX*, May/Jun 2004, pp 35 – 38.

[3]Robert J. Zavrel Jr, W7SX, "Maximizing Radiation Resistance in Vertical Antennas," *QEX*, Jul/Aug 2009, pp 28 – 33.